

Feasibility of electron cyclotron autoresonance acceleration by a short terahertz pulse

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A vacuum autoresonance accelerator scheme for electrons, which employs terahertz radiation and currently available magnetic fields, is suggested. Based on numerical simulations, parameter values, which could make the scheme experimentally feasible, are identified and discussed.

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The aim of this Letter is to investigate conditions for an electron vacuum autoresonance accelerator scheme that would employ circularly polarized terahertz (THz) radiation (or T-rays) and currently available laboratory magnetic fields, with the hope of stimulating future experiments. The system, subject of this Letter, is an electron (or electron bunch) injected in the common directions of radiation pulse propagation and an added uniform magnetic field (see Fig. 1).

Autoresonance Laser Acceleration (ALA) of electrons has a relatively long history [1–6]. Theoretical vacuum ALA studies [4, 7] have shown that the electron would gain a tremendous amount of energy from the laser field when the initial injection energy, the laser frequency ω and the external magnetic field strength B_s , all conspire to achieve resonance, or near-resonance. For axial injection, and plane-wave fields, the resonance condition takes the form

$$r \equiv \frac{\omega_c}{\omega_D} = \frac{eB_s}{m\omega} \sqrt{\frac{1+\beta_0}{1-\beta_0}} \rightarrow 1, \quad (1)$$

where, $\omega_c = eB_s/m$ is the cyclotron frequency of the electron around the lines of the static magnetic field, and ω_D is the Doppler-shifted frequency of the radiation field, as seen by the electron. Furthermore, m and e are the electron's mass and charge, respectively, and SI units have been used. Also, β_0 is the speed with which the electron is injected, scaled by the speed of light c . Autoresonance essentially means the electron cyclotron frequency becomes comparable to the Doppler-shifted frequency, seen by the electron, of the (circularly polarized) laser. When this happens, the velocity vector β of the electron and the electric field vector \mathbf{E} of the laser field maintain the same angle during interaction, which leads, according to the equation

$$\frac{d\varepsilon}{dt} = -ec\beta \cdot \mathbf{E}, \quad (2)$$

to synchronous energy gain by the particle from the radiation field. In Eq. (2) the electron's relativistic energy is $\varepsilon = \gamma mc^2$, where $\gamma = (1 - \beta^2)^{-1/2}$. On resonance, the vectors β and \mathbf{E} gyrate, about the common direction

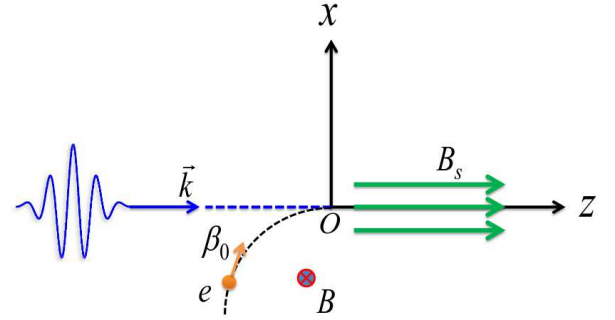


FIG. 1. (Color online) Schematic of a possible ALA setup. A magnetic field B is used to bend the electron beam for collision with the THz pulse, which propagates along $+z$ (propagation vector \vec{k}). Front of the pulse catches up with the electron precisely at the origin of coordinates O . The magnetic field $B_s = B_s \hat{k}$ is responsible for the electron cyclotron motion.

of the magnetic field and laser propagation, at the same frequency. According to Eq. (2), the field-to-electron energy transfer rate is a maximum when β and \mathbf{E} are antiparallel.

To the best of our knowledge, a vacuum autoresonance laser accelerator has not been realized and none is currently available and in operation. Plane-wave-based studies have shown that, for the scheme to work, a strong magnetic field needs to be maintained over a long distance, which makes such a device both expensive and prohibitively too long. Hope has been revived by recent investigations, in which petawatt optical-frequency laser pulses, modeled most realistically [8] by Gaussian fields, have been employed. It has been shown that electrons can gain over 10 GeV of energy in a magnetic field of strength exceeding 50 T, and maintained constant over a distance of several meters [8]. Unfortunately, even these conditions are at present extremely difficult to realize.

In this Letter, the laser fields will be replaced by THz fields, and the parameter space will be scanned in search of a parameter set which may be more likely to be experimentally realized in the near future. According to the condition (1) vacuum autoresonance may be achieved in a uniform magnetic field of a few tesla, provided the radia-

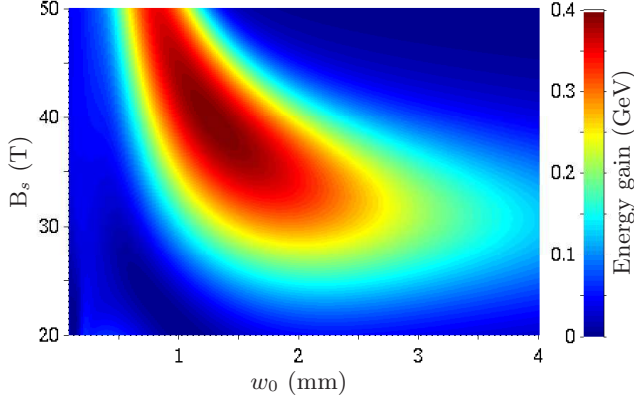


FIG. 2. (Color online) Contour plot of the energy gained by a single electron, injected with $\gamma_0 = 3$ for interaction with a single circularly polarized Gaussian pulse, as functions of the magnetic field strength B_s and the waist radius at focus w_0 . The pulse power is $P = 100$ TW, and its frequency is $f = 4$ THz ($\lambda = 75 \mu\text{m}$, period $T_0 = 250$ fs = FWHM).

tion frequency is lowered by roughly two orders of magnitude. Lowering the frequency by two orders of magnitude from the optical domain lands one in the THz region of the electromagnetic spectrum, roughly in the range 0.3 – 10 THz.

Low-power THz radiation has been the subject of intense investigation for many years now, and applications in such fields as imaging, wireless communication and remote sensing, have seen much progress recently [9–14]. In pondering the idea of utilizing THz radiation in a vacuum ALA-like scheme, one must immediately come to grips with the need for high power. Currently considered the best source for high-power THz radiation, a free-electron laser (FEL) can generate radiation of only a few gigawatt (GW) power [10, 14]. Furthermore, promising candidates for generating more intense T-rays are the interactions of high intensity lasers with plasma or gas targets [15–17]. Thus, to experimentally realize a vacuum ALA-like THz scheme, the only remaining (and admittedly non-trivial) challenge would be to develop sources of THz radiation of power in the terawatt (TW) region and beyond, as will be demonstrated shortly.

The single- and many-particle simulations in this Letter are all based on solving the relativistic energy-momentum transfer equations

$$\frac{d\boldsymbol{\beta}}{dt} = \frac{e}{\gamma mc} [\boldsymbol{\beta}(\boldsymbol{\beta} \cdot \mathbf{E}) - (\mathbf{E} + c\boldsymbol{\beta} \times \mathbf{B})], \quad (3)$$

for the electron, subject to the adopted initial conditions of axial injection with scaled energy $\gamma_0 = (1 - \beta_0^2)^{-1/2}$, together with the assumption that the front of the pulse catches up with the particle at $t = 0$, precisely at the origin of coordinates. In Eq. (3), \mathbf{B} is the sum of the radiation magnetic field and \mathbf{B}_s . Fields of the circularly polarized THz radiation are modeled by those of a short Gaussian pulse [18, 19].

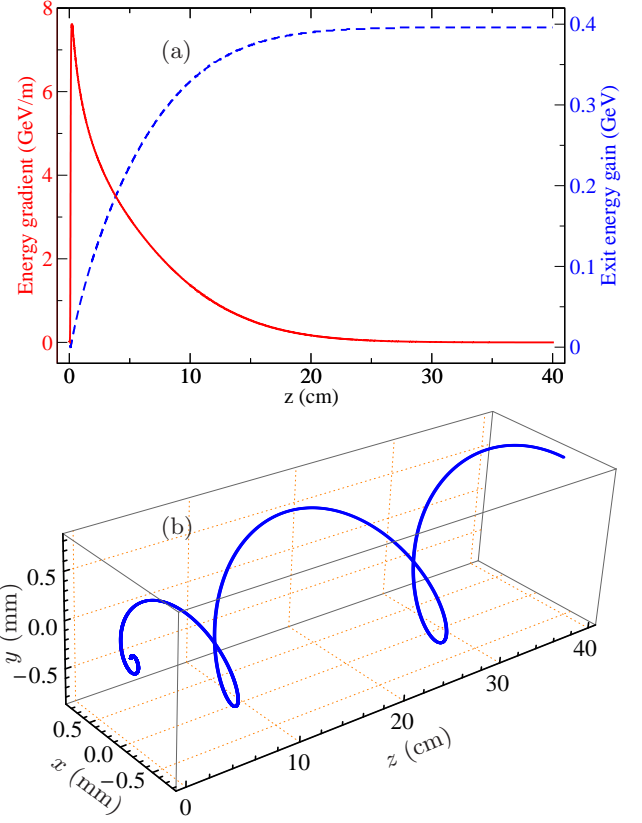


FIG. 3. (Color online) (a) Single electron energy gain (dashed blue) and energy gradient (solid red) as functions of its forward excursion distance. The parameters are those of Fig. 2, in addition to $w_0 = 17\lambda \approx 1.27$ mm, and $B_s \approx 39.6$ T. (b) Actual trajectory of the electron under the conditions of (a).

In addition to the magnetic field strength B_s and the THz frequency ω , a third parameter, namely β_0 and, hence, the scaled injection energy of the electron γ_0 , plays a decisive role in determining the (exact, plane-wave-based) resonance condition (1). When a more realistic model is adopted, such as that of a Gaussian beam or pulse, detuning away from resonance results, and one is forced to search for near-resonance by scanning the parameter space of Fig. 2, for example [8].

For further discussion, we select from Fig. 2 a parameter set which leads to near-autoresonance and, hence, high energy gain. With the gain defined by

$$G = (\gamma - \gamma_0)mc^2, \quad (4)$$

Fig. 3(a) shows that an electron exits with kinetic energy $K_{exit} \sim 396.2601$ MeV from interaction with a single-cycle, 100 TW pulse, over a distance of less than 40 cm, and in the presence of a magnetic field of strength 39.6 T. Note that most of the energy is gained over nearly the first 20 cm, and very little energy is gained beyond that point as the electron interacts with the weak tail of the pulse and is ultimately left behind. The energy gradient,

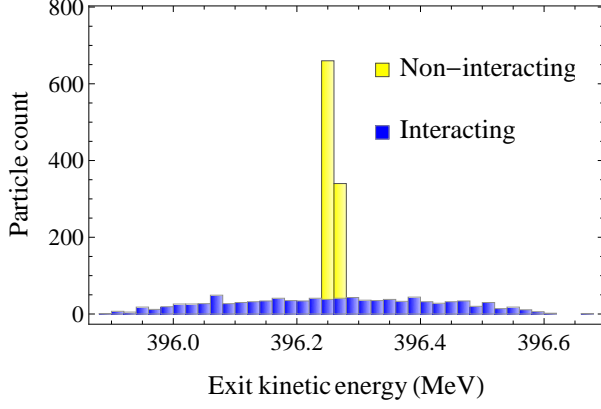


FIG. 4. (Color online) Exit kinetic energy distribution of a 1000 ensemble of electrons. Interacting: Coulomb electron-electron interactions are turned on, and Non-interacting: Coulomb interactions are turned off. Electrons are assumed, initially, to be distributed randomly within a cylinder of radius $0.232 \mu\text{m}$ and height $4.642 \mu\text{m}$, and centered at the origin of coordinates. Electron kinetic energy is, initially, distributed normally, with mean $\bar{K}_0 = 1.022 \text{ MeV}$ and spread $\Delta K_0 = 0.1\%$.

or the gain per unit forward excursion distance,

$$\frac{dG}{dz} = -e \left(\frac{\beta \cdot \mathbf{E}}{\beta_z} \right), \quad (5)$$

peaks over the first one centimeter and falls down to zero very quickly. The peak gradient is almost 8 GeV/m , or about 80 times the fundamental limit on the performance of conventional accelerators [20]. An *average gradient*, found simply by dividing the total gain by the total forward excursion during interaction with the pulse, is about 1 GeV/m , in this example.

Figure 3(b) shows the trajectory of the electron whose gain and gradient are shown in (a). The trajectory is the expected helix of increasing cross section [4]. Note that the transverse dimensions of the helical trajectory are much smaller than the longitudinal electron excursion, which makes the trajectory essentially linear.

The discussion so far has been limited to THz – ALA of a single electron. Next, we consider acceleration of an ensemble of 1000 electrons randomly distributed, initially, within a cylinder centered at the origin of coordinates, and whose radius and height are $0.232 \mu\text{m}$ and $4.642 \mu\text{m}$, respectively. Initial kinetic energy of the ensemble has a normal distribution of mean $\bar{K}_0 = 1.022 \text{ MeV}$ and spread $\Delta K_0 = 0.1\%$. For the parameter set used in Fig. 3, our simulations yield the exit energy distributions displayed in the histograms of Fig. 4. From the data, one gets $\bar{K}_{exit} = 396.2583 \pm 0.0028 \text{ MeV}$ (or a spread of 0.0007%), with the electron-electron Coulomb interactions turned off, and $\bar{K}_{exit} = 396.2562 \pm 0.1682 \text{ MeV}$ (0.042%), when those Coulomb interactions are properly taken into account. These many-particle results agree quite well with our single-particle calculations (Fig.

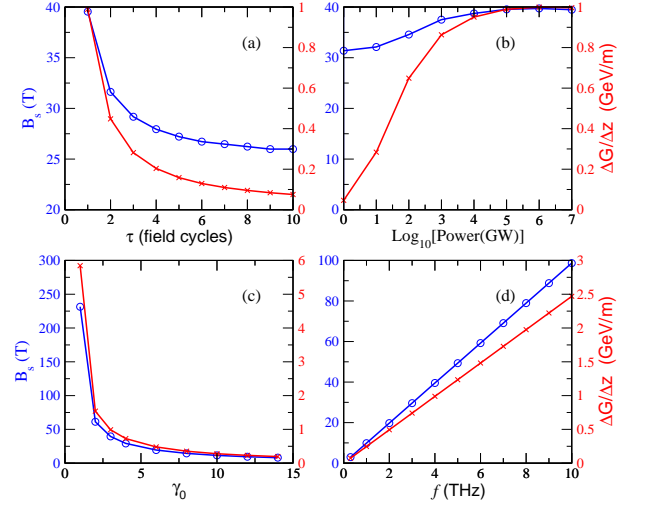


FIG. 5. (Color online) Near-resonance magnetic field dependence (blue and circles) upon: (a) the THz pulse duration τ in units of the radiation field cycle T_0 , (b) the THz power, (c) the scaled injection energy γ_0 , and (d) the THz frequency. For each data point in (a) – (d) the average energy gradient in GeV/m is given (red and crosses). Each data point is a result of calculations, along the lines of the work which led to Figs. 2 and 3, and employing their (other) parameters. For example, in (a) $\gamma_0 = 3$, the power is 100 TW , $w_0 = 17 \lambda$, and $f = 4 \text{ THz}$, and so on.

3, $K_{exit} \sim 396.2601 \text{ MeV}$). Note also that the particle-particle interactions do result in a noticeable increase in the spread in exit energies. In absolute terms, however, the effect of incorporating the Coulomb interactions is small, due to the fact that the particle density of the initial ensemble is quite low ($\sim 1.273 \times 10^{21} \text{ m}^{-3}$).

Our focus in this Letter has been to demonstrate that THz – ALA of electrons may be experimentally feasible with present-day technology, as far as the needed magnetic field strengths are concerned. However, the example presented above in some detail required the use of $B_s \sim 40 \text{ T}$. This kind of field strength is available only at large and expensive facilities [21, 22]. For weaker fields to be utilized, other parameters have to be compromised. Values have to be picked for four more parameters which may ultimately lead to energy gains and energy gradients more modest than has been reported above. Figures 5(a) – (d) show how the values of B_s , corresponding to near-resonance, vary with the pulse duration, the power of the THz radiation source, the scaled injection energy, and the THz frequency, respectively. Shown in red also are the corresponding average energy gradients, defined in each case as the total exit energy gain divided by the total forward excursion distance, during interaction with the pulse. The results displayed in Fig. 5 emerge from calculations following along the same lines leading to Figs. 2 and 3, for each set of parameters separately. In most cases considered, the average energy gradients are much higher than the natural limit on performance of the conventional accelerators, namely, about 100 MeV/m . Note

that in calculating these average gradients, the full excursion distance occurring during the full particle-field interaction time, is used. Considering that most of the energy is gained during interaction with only a small fraction of the excursion, leads to the conclusion that the energy gradients are, effectively, much higher than is suggested by the averages reported in Fig. 5.

Figure 5(a) shows clearly that the use of currently available laboratory magnetic fields in a THz – ALA setup is feasible, but the energy gradients that would be achieved fall to just a few times the performance limit of conventional accelerators [20]. Recall that the example considered above (Fig. 3) made use of a one-cycle pulse. With several-cycle pulses, the resonance magnetic field strengths required fall already below 30 T, as shown in Fig. 5(a). The same conclusions may be drawn from Fig. 5(c). However, the opposite trend exhibited in Figs. 5(b) and (d) confirms earlier conclusions. Increasing the source power and frequency calls for the need for stronger resonance magnetic fields, and leads to high gains and

ultra-high energy gradients [4, 8]. On-resonance dependence of B_s upon the frequency f is linear in Fig. 5(d), essentially like the plane-wave case (1).

In conclusion, acceleration by cyclotron autoresonance of electrons in the simultaneous presence of THz fields and a uniform magnetic field of strength currently available for laboratory experiments, is feasible, provided a THz radiation source of TW power is also available. This is the case when the electrons are injected along the common directions of magnetic field and radiation propagation. According to Fig. 5(c), even acceleration from rest is possible, but the magnetic field strength needed for such a scheme is about 300 T, in this case.

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